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Chance-Constrained Stochastic Model Predictive Control for Managed Pressure Drilling with Dissolution-Aware Thermodynamic State Augmentation

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Abstract

Managed pressure drilling has expanded the feasible operating envelope in wells with narrow pressure margins by enabling active manipulation of surface backpressure and flow-out. However, the same closed-loop actuation that stabilizes bottomhole pressure also complicates diagnosis and mitigation of gas influx events, because pressure and pit-volume signatures become entangled with control actions, compressibility, and temperature-dependent fluid properties. A persistent source of ambiguity is the partitioning of influx gas between dissolved and free phases, which can produce liquid swelling and alter volumetric returns without immediate appearance of a distinct free-gas holdup. This paper develops a risk-aware control architecture that explicitly embeds dissolution-driven swelling into the drilling hydraulics state, enabling proactive mitigation through stochastic model predictive control with chance constraints. The technical contribution is a tractable, differentiable control-oriented model that couples one-dimensional annular mass and momentum balances with a thermodynamic dissolution closure and a bounded, stable discretization suitable for real-time optimization. Uncertainty in fluid properties, frictional pressure losses, sensor bias, and influx parameters is represented through a structured stochastic parametrization. A scenario-based approximation of probabilistic safety constraints is then integrated into a receding-horizon optimizer that selects choke and pump trajectories to minimize operational deviation while keeping bottomhole pressure and surface handling risk within prescribed probabilities. The resulting formulation supports joint inference and control by maintaining consistent uncertainty propagation, and it provides explicit certificates of risk for impending surface gas handling. Numerical experiments demonstrate that dissolution-aware state augmentation reduces spurious conservatism relative to free-gas-only models and improves feasibility in regimes where swelling masks early influx indicators.

1. Introduction

Drilling operations are increasingly conducted in regimes where the allowable window between pore pressure and fracture pressure is narrow, and the hydraulic system is operated close to its limits to maintain rate of penetration and wellbore stability [1]. In such regimes, the difference between a benign transient and a hazardous event may be minutes, and decision-making is often performed under incomplete information. Managed pressure drilling provides a mechanism for rapid actuation through surface backpressure and controlled returns, but it does not remove the need for reliable interpretation of what the well is doing; it changes the inference problem because actuation alters the signals that would otherwise reveal an influx. As a result, a purely diagnostic algorithm that looks for anomalous pit gain or pressure drop can be insufficient, because these anomalies can be induced by deliberate choke changes, pump ramps, or rheology shifts, and because early-stage influx can be partly hidden by thermodynamic absorption of gas into the drilling fluid.

A common operational heuristic associates pit gain with free-gas volume entering the annulus. Yet, for oil-based and synthetic-based drilling fluids, a substantial fraction of gas may dissolve, particularly at elevated pressure, causing the liquid phase to swell and changing sur-

Symbol	Description	Units	Role in model
$p(z,t)$	Mixture pressure along the annulus	Pa or psi	State and safety output
$u_m(z,t)$	Mixture superficial velocity	m/s	Convective transport state
$\alpha_g(z,t)$	Free-gas volume fraction	–	Phase inventory state
$x_d(z,t)$	Dissolved gas mass fraction in liquid	–	Thermodynamic composition state
$x_d^\star(p,T)$	Equilibrium dissolved gas fraction	–	Thermodynamic target for dissolution
$B_l(x_d,p,T)$	Swelling factor modifying liquid density	–	Links dissolution to density and pit gain
$T(z,t)$	Temperature profile along the annulus	$^\circ$C or K	Influences solubility and compressibility

Table 1: Key state and thermodynamic variables used in the control-oriented annular hydraulics model.

Relation	Physical meaning	Notes
$\partial_t(\alpha_g\rho_g)+\partial_z(\alpha_g\rho_g u_g)=s_k(z,t)-\Gamma_d$	Free-gas mass balance with influx and dissolution sink	Localized kick source s_k
$\partial_t(\alpha_l\rho_l x_d)+\partial_z(\alpha_l\rho_l x_d u_l)=\Gamma_d$	Dissolved gas transport and accumulation	Couples to swelling and returns
$\partial_t(\rho_m u_m)+\partial_z(\rho_m u_m^2+p)=-\rho_m g-\lambda_f(\theta_f)\rho_m u_m\lvert u_m\rvert$	Mixture momentum balance with gravitational and frictional forces	Control-oriented friction surrogate
$\rho_g(p,T)=\dfrac{pM_g}{Z(p,T)RT}$	Real-gas density closure	Uses tabulated $Z(p,T)$
$\rho_l(p,T,x_d)=\dfrac{\rho_{l,0}(T)}{B_l(p,T,x_d)}$	Swelling-modified liquid density	Bounded, smooth B_l
$\Gamma_d=\kappa_d\alpha_l\rho_l\big(x_d^\star(p,T)-x_d\big)$	Relaxation-based dissolution rate	Effective mass-transfer kinetics

Table 2: Principal governing equations in the control-oriented hydraulics and dissolution model.

face volumetric returns without an equivalent free-gas holdup at the same time [2]. This mechanism alters both the time scale and the magnitude of observable signals, and it can render free-gas-only models overly conservative or misleading. In particular, an early pit gain caused primarily by swelling can be misinterpreted as an imminent arrival of free gas at surface, which may trigger unnecessary shut-in or overly aggressive backpressure schedules that degrade operational efficiency or introduce other hazards.

Thermodynamic modeling has clarified how dissolution and swelling can be predicted from state variables such as pressure and temperature. Manikonda et al. (2020) [3] introduced first-of-a-kind thermodynamic models aimed at estimating gas-kick dissolution effects in oil-based drilling fluids, linking solubility to measurable operational conditions and to volumetric behavior relevant to pit gain interpretation. Relatedly, Manikonda et al [4]. (2021) presented a thermodynamic modeling approach for mud swelling under gas-kick conditions [5], providing a mechanistic pathway for representing how dissolved gas can modify effective liquid properties during migration. These developments are relevant here not as the foundation of the paper, but as a motivating indication that dissolution and swelling can be embedded as state-dependent mechanisms rather than treated as ad hoc corrections.

The central thesis of this paper is that the operational objective should be formulated as a risk-aware control problem rather than a detection-only problem. From a systems viewpoint, managed pressure drilling is a controlled dynamical system with constraints. The control inputs, such as choke opening and pump rate, alter annu-

Symbol / term	Description	Source of uncertainty
θ_f	Effective friction parameters in $\lambda_f(\theta_f)$	Rheology changes, cuttings loading, roughness
θ_s	Slip and drift parameters in $V_s(\alpha_g, p, T; \theta_s)$	Flow pattern transitions, empirical correlations
κ_d	Effective dissolution mass-transfer coefficient	Scale-up from lab data, fluid formulation variability
$s_k(z, t)$	Influx source profile (depth, onset, rate)	Unknown reservoir connectivity and kick characteristics
Sensor biases (e.g. b_p, b_q)	Bias in pressure and flow measurements	Calibration drift, sensor aging
Property correlations (e.g. $Z(p, T)$, B_l coefficients)	Thermodynamic and swelling relationships	Limited measurements, regression error

Table 3: Uncertain parameters and disturbances represented in the stochastic model for managed pressure drilling.

Component	Mathematical form (per stage)	Operational role
Pressure tracking	$\left\| p_{\text{bh}}(x_k) - p_{\text{ref}} \right\|_{Q_p}^2$	Keep bottomhole pressure within the target window
Actuation effort	$\left\| u_k - u_{\text{ref}} \right\|_{Q_u}^2$	Penalize aggressive choke and pump moves
Gas-handling penalty	$\phi_{\text{gas}}(y_k)$	Discourage high surface gas fraction or gas flow
Pit/returns penalty	$\phi_{\text{pit}}(V_{\text{pit}}(x_k))$	Limit pit volume excursions and return-rate spikes
Terminal cost	$\ell_f(x_H)$	Shape end-of-horizon state and improve stability

Table 4: Main components of the stochastic MPC stage cost used for influx mitigation and operational performance.

lar pressure distribution and the transport of phases; the disturbances include uncertain influx, uncertain rheology, and uncertain property correlations; and the safety constraints include maintaining bottomhole pressure within a window and keeping surface gas handling within manageable limits. A control policy that explicitly accounts for uncertainty and for dissolution-driven swelling can reduce both false alarms and unnecessary conservatism by distinguishing between hazards that are immediate, hazards that are probable but delayed, and benign transients that mimic hazards [6].

This paper proposes a chance-constrained stochastic model predictive control architecture in which dissolution-driven swelling is represented as an augmented thermodynamic state. The controller uses a control-oriented model that is differentiable and stable, enabling real-time optimization under uncertainty. The optimization minimizes a composite objective capturing deviation from desired operating conditions, actuation effort, and predicted risk of unacceptable surface gas volume fraction and bottomhole pressure excursions. Safety is enforced probabilistically through chance constraints, approximated via a scenario method with adaptive scenario allocation. The resulting controller can operate in a dual mode: it can mitigate a suspected influx by adjusting backpressure while simultaneously shaping the system excitation to improve identifiability of the influx state, thereby reducing uncertainty over time.

The technical contributions are threefold [7]. First, the paper develops a consistent state-space formulation for annular hydraulics with a dissolution and swelling closure that is bounded, numerically stable, and suitable for embedding inside an optimizer. Second, it derives a chance-constrained MPC formulation that couples uncertainty

Quantity	Chance-constraint form	Bound parameters	Risk level
Bottomhole pressure	$\mathbb{P}(p_{\text{min}} \leq p_{\text{bh}}(x_k) \leq p_{\text{max}}) \geq 1 - \epsilon_p$	$p_{\text{min}}, p_{\text{max}}$	ϵ_p
Surface gas flow	$\mathbb{P}(q_g(x_k) \leq q_{g,\text{max}}) \geq 1 - \epsilon_g$	$q_{g,\text{max}}$	ϵ_g
Surface gas fraction	$\mathbb{P}(\alpha_{g,\text{surf}}(x_k) \leq \alpha_{g,\text{max}}) \geq 1 - \epsilon_\alpha$	$\alpha_{g,\text{max}}$	ϵ_α
Pit volume	$\mathbb{P}(V_{\text{pit}}(x_k) \leq V_{\text{max}}) \geq 1 - \epsilon_v$	V_{max}	ϵ_v
Equivalent circulating density	$\mathbb{P}(\text{ECD}(x_k) \leq \text{ECD}_{\text{max}}) \geq 1 - \epsilon_{\text{ECD}}$	ECD_{max}	ϵ_{ECD}

Table 5: Safety-related chance constraints enforced in the stochastic MPC formulation for managed pressure drilling.

Element	Symbol	Description
Prediction horizon	H	Number of discrete steps over which the dynamics are optimized
Scenario count	S	Number of disturbance and parameter realizations used per MPC update
Disturbance sequences	$\{w_{0:H-1}^{(s)}\}$	Sampled realizations of influx, friction, slip, and dissolution parameters
State trajectories	$\{x_k^{(s)}\}$	Scenario-specific rollouts under shared control sequence $\{u_k\}$
Control sequence	$\{u_0, \ldots, u_{H-1}\}$	Choke and pump moves optimized jointly across all scenarios
Scenario allocation	$S_p, S_g, \ldots$	Adaptive focus of samples on pressure, gas-handling, or volume risk drivers

Table 6: Configuration elements of the scenario-based chance-constrained MPC used in the proposed framework.

propagation with safety constraints relevant to influx management, including constraints on bottomhole pressure, equivalent circulating density, and predicted surface gas handling. Third, it provides a differentiable reduced-order representation that preserves essential constraints such as positivity of densities and phase fractions, enabling efficient gradient-based optimization without violating physical feasibility.

The paper is organized into five technical sections following this introduction. The next section develops the control-oriented dynamical model, emphasizing bounded closures and discretization choices that preserve feasibility [8]. The subsequent section formulates the stochastic MPC problem with chance constraints and discusses scenario approximations and computational structure. The next section introduces the differentiable surrogate and sensitivity computation needed for real-time optimization. The following section develops uncertainty propagation, state estimation, and dual control considerations that arise when influx parameters are unknown. The final technical section reports numerical experiments illustrating risk reduction and feasibility improvement in dissolution-dominated regimes. The paper ends with a conclusion highlighting implications and limitations [9].

2. Control-Oriented Hydraulics and Thermodynamic State Augmentation

A controller embedded in managed pressure drilling must compute actions faster than the time scale on which the hydraulic system evolves, and it must do so under uncertain states and parameters. This necessitates a model that is physically grounded but computationally tractable. The model developed here is one-dimensional along measured depth, representing the annulus as a distributed parameter system that is discretized into a finite-dimensional state. The intent is not to resolve detailed multiphase flow patterns, but to provide a control-relevant mapping from inputs to safety-relevant outputs, while explicitly representing dissolution and swelling as mechanisms that shape observations and constraints.

Let $z \in [0, L]$ denote the annular coordinate from bit to surface [10]. The state includes mixture pressure $p(z,t)$, mixture superficial velocity $u_m(z,t)$, free-gas volume fraction $\alpha_g(z,t)$, and a dissolved-gas scalar $x_d(z,t)$ that represents dissolved mass fraction in the liquid. The state is augmented by a thermodynamic equilibrium target $x_d^\star(p,T)$ computed from a pre-tabulated mapping, and by a swelling factor $B_l(x_d, p, T)$ that modifies the effective liquid density and thus hydrostatic head and volumetric returns. Temperature is represented as a slowly varying

Component	Symbol / map	Design choice	Purpose
Encoder	$E : x \mapsto z$	Structured map with log and sigmoid coordinates	Compress full state while respecting sign and bounds
Decoder	$D : z \mapsto \tilde{x}$	Positivity- and boundedness-preserving reconstruction	Enforce physical feasibility for all latent states
Latent dynamics	$F(z, u, \theta)$	Low-order polynomial and bilinear operator	Fast, differentiable prediction for MPC rollouts
Thermodynamic embedding	$x_d^\star(p, T)$, $Z(p, T)$	Tabulated, differentiable interpolation layers	Retain realistic solubility and compressibility trends
Conservation regularization	$m_g(\tilde{x})$	Penalize gas inventory mismatch in training	Maintain consistent total gas mass evolution
Uncertainty input	θ	Low-dimensional parameter vector in dynamics	Represent friction, slip, and dissolution uncertainty

Table 7: Design features of the differentiable reduced-order model used as the prediction core in the controller.

Scenario class	Dominant physical effects	Evaluation focus
Dissolution-dominated influx	High solubility and fast κ_d; pronounced swelling and delayed free gas	Feasibility and conservatism when pit gain is driven by swelling
Free-gas-dominated influx	Low solubility or slow κ_d; persistent mobile free gas	Surface gas-handling risk and bottomhole pressure containment
Operational transients without influx	Pump ramps, choke changes, rheology shifts	False positives and unnecessary actuation under benign transients
Parameter misspecification	Biased friction, slip, and dissolution parameters	Robustness of chance constraints to modeling errors
Narrow-window wells	Tight pore–fracture margins and equipment limits	Trade-off between bottomhole safety and operational efficiency

Table 8: Representative numerical scenario classes used to assess the proposed control architecture.

profile $T(z,t)$ supplied by a low-order evolution model or by a schedule, because the controller must account for temperature dependence of solubility and density without adding excessive stiffness or unobserved dynamics.

The distributed mass balance for the gas inventory is separated into free and dissolved components. The free-gas mass per unit volume is $\alpha_g \rho_g$, and the dissolved-gas mass per unit volume is $\alpha_l \rho_l x_d$ with $\alpha_l = 1 - \alpha_g$. A conservative formulation with interconversion reads [11]

$$\begin{aligned} \partial_t(\alpha_g \rho_g) + \partial_z(\alpha_g \rho_g u_g) &= s_k(z,t) - \Gamma_d \\ \partial_t(\alpha_l \rho_l x_d) + \partial_z(\alpha_l \rho_l x_d u_l) &= \Gamma_d \end{aligned} \qquad (1)$$

where $s_k(z,t)$ represents influx mass source localized near the kick depth when present, and Γ_d is the net dissolution rate. Liquid mass balance is expressed similarly but is typically dominated by imposed circulation and is coupled through swelling-modified density.

A control-oriented momentum balance is written for the mixture with an effective friction closure. Define mixture density $\rho_m = \alpha_g \rho_g + \alpha_l \rho_l$ and mixture velocity $u_m = \alpha_g u_g + \alpha_l u_l$. A simplified compressible momentum equation is [12]

$$\partial_t(\rho_m u_m) + \partial_z(\rho_m u_m^2 + p) = -\rho_m g - \lambda_f(\theta_f)\, \rho_m u_m |u_m| \qquad (2)$$

where $\lambda_f(\theta_f)$ is an effective friction coefficient parametrized by uncertain parameters θ_f capturing rheology and roughness. The quadratic drag form is used as a control surro-

Area	Contribution	Related sections
Hydraulics and thermodynamics model	One-dimensional annular model with dissolution-aware swelling state and bounded closures	Sec. Control-Oriented Hydraulics and Thermodynamic State Augmentation
Chance-constrained MPC	Risk-aware formulation with probabilistic limits on pressure, gas handling, and volume	Sec. Chance-Constrained Stochastic Model Predictive Control Formulation
Reduced-order surrogate	Differentiable reduced-order model preserving positivity and bounds for optimization	Sec. Differentiable Reduced-Order Prediction Model for Real-Time Optimization
Uncertainty and dual control	Estimation and information-seeking control under uncertain influx and parameters	Sec. Uncertainty Propagation, Estimation, and Dual Control Considerations
Numerical evaluation	Comparison of dissolution-aware and free-gas-only controllers across regimes	Sec. Numerical Evaluation in Dissolution-Dominated and Free-Gas-Dominated Regimes

Table 9: Summary of main methodological contributions and where they appear in the manuscript.

gate; it can approximate laminar-to-turbulent transitions through parameter variation and can be calibrated against steady hydraulics calculations.

The gas density is modeled as a real-gas function of pressure and temperature,

$$\rho_g(p,T) = \frac{pM_g}{Z(p,T)RT} \tag{3}$$

where $Z(p,T)$ is a compressibility factor tabulated offline. The liquid density is modified by swelling, [13]

$$\rho_l(p,T,x_d) = \frac{\rho_{l,0}(T)}{B_l(p,T,x_d)} \tag{4}$$

with B_l chosen to be bounded below by 1 and to remain smooth for differentiability. The swelling factor is expressed as a low-order function of x_d with mild dependence on pressure and temperature to reflect state dependence without expensive evaluation. A representative form is

$$B_l = 1 + \beta_1 x_d + \beta_2 x_d^2 \tag{5}$$

where coefficients are treated as uncertain but bounded.

The dissolution kinetics are represented by relaxation to equilibrium,

$$\Gamma_d = \kappa_d\, \alpha_l \rho_l \big(x_d^\star(p,T) - x_d\big) \tag{6}$$

where κ_d is an effective mass-transfer coefficient [14]. This representation is control-oriented: it captures the principal time scale separating instantaneous equilibrium from frozen composition, and it allows the controller to reason about whether a pressure change will quickly dissolve free gas or will predominantly expand it.

Slip between phases influences transport time and surface arrival, yet a full two-fluid model is often unnecessary for control decisions at the surface. The model uses a drift relation for the gas velocity,

$$u_g = u_m + V_s(\alpha_g, p, T; \theta_s) \tag{7}$$

with V_s a slip velocity closure. The liquid velocity follows from mixture velocity and phase fractions [15]. The slip parameters θ_s are uncertain, and their uncertainty is propagated by the controller to prevent overconfident predictions of surface gas arrival.

Boundary conditions represent managed pressure drilling actuation. The primary control inputs are choke actuation, represented through a surface pressure condition, and pump rate, represented through an inlet flow condition. A simplified choke relation is

$$p(L,t) = p_{\text{sep}} + \Delta p_{\chi}\big(q_{\text{out}}(t), u_{\chi}(t)\big) \tag{8}$$

where $u_{\chi}(t)$ is the choke control signal and p_{sep} is downstream separator pressure. The pump rate sets the inlet mixture flow; changes propagate downhole through compressibility and inertia [16]. The safety-relevant output of bottomhole pressure is $p(0,t)$, and equivalent circulating density can be computed from pressure gradient.

For embedding in MPC, the distributed equations are discretized into N cells with a conservative finite-volume method. The choice of discretization is constrained by the need to preserve positivity of densities and phase fractions. A positivity-preserving flux scheme is used for advective terms, and the dissolution reaction is updated implicitly to avoid stiffness. Specifically, after an explicit transport update produces intermediate x_d, an implicit reaction step yields [17]

$$x_d^{n+1} = \frac{x_d^n + \Delta t\, \kappa_d x_d^{\star}}{1 + \Delta t\, \kappa_d} \tag{9}$$

ensuring stability for large κ_d. Phase fraction updates are projected through bounded coordinate transforms so that $\alpha_g \in [0,1]$ is maintained.

The resulting discrete-time state update can be written compactly as

$$x_{k+1} = f(x_k, u_k, \xi_k) \tag{10}$$

where $x_k \in \mathbb{R}^n$ is the discretized state, u_k collects choke and pump inputs, and ξ_k collects uncertain parameters and disturbances such as influx. This model is the basis for prediction in MPC. It is deliberately structured so that the uncertainty enters through a small set of interpretable parameters, enabling tractable uncertainty propagation and scenario generation.

A key output for surface risk is the predicted gas volumetric fraction at surface and the predicted gas handling rate, both of which depend on how much gas is free versus dissolved [18]. Dissolution-aware modeling alters these predictions: increased dissolved fraction reduces immediate free-gas handling risk but can increase liquid swelling and thus pit gain and surface return volume. The controller must therefore trade off constraints that are traditionally treated independently, such as pit volume management and surface gas handling. The next section formalizes this trade-off as a risk-aware optimization problem.

3. Chance-Constrained Stochastic Model Predictive Control Formulation

Model predictive control computes a sequence of future control moves by minimizing an objective subject to predicted dynamics and constraints, then applies only the first move and repeats the computation at the next time step. In managed pressure drilling, MPC is appealing because constraints are central: bottomhole pressure must remain between pore and fracture limits, choke pressure must remain within hardware limits, and surface gas handling must remain below equipment capacity [19]. However, both states and model parameters are uncertain, particularly during suspected influx. A deterministic constraint based on a nominal model can be either unsafe if the nominal is optimistic or overly conservative if the nominal ignores dissolution-driven buffering. Chance constraints provide a principled compromise by enforcing that constraints hold with a specified probability.

Let the discrete-time prediction model be

$$\begin{aligned} x_{k+1} &= f(x_k, u_k, w_k) \\ y_k &= h(x_k, u_k) \end{aligned} \tag{11}$$

where w_k represents stochastic disturbances and uncertain parameters, and y_k collects outputs such as bottomhole pressure, surface return flow, and surface gas fraction [20]. The controller operates on an estimated state $\hat{x}_0$ with an uncertainty distribution, and it predicts distributions of future states given candidate controls.

The MPC objective balances operational goals and control effort. A representative stage cost is

$$\ell(x_k, u_k) = \|p_{\text{bh}}(x_k) - p_{\text{ref}}\|_{Q_p}^2 + \|u_k - u_{\text{ref}}\|_{Q_u}^2 + \phi_{\text{gas}}(y_k) \tag{12}$$

where p_{bh} is bottomhole pressure, p_{ref} is a desired setpoint within the window, and ϕ_{gas} penalizes predicted surface gas handling. The gas penalty is chosen to be smooth so that gradients exist, but it is weighted relative to constraints to avoid encouraging the optimizer to violate safety.

Constraints include deterministic bounds on actuators and probabilistic constraints on safety variables. Deterministic constraints are

$$u_{\text{min}} \le u_k \le u_{\text{max}} \tag{13}$$

including choke position and pump rate bounds and rate-of-change limits [21].

Safety constraints are formulated probabilistically. For bottomhole pressure, define lower and upper bounds p_{min} and p_{max}. A chance constraint is

$$\mathbb{P}\Big(p_{\text{min}} \le p_{\text{bh}}(x_k) \le p_{\text{max}}\Big) \ge 1 - \epsilon_p \tag{14}$$

where ϵ_p is an allowable violation probability. Similarly, for surface gas handling, define a maximum allowable free-gas volumetric flow $q_{g,\text{max}}$ or maximum gas fraction at surface $\alpha_{g,\text{max}}$. A chance constraint is

$$\mathbb{P}\Big(q_g(x_k) \le q_{g,\text{max}}\Big) \ge 1 - \epsilon_g \tag{15}$$

These constraints reflect operational risk tolerance and can be tuned to match equipment reliability and hazard severity [22].

In nonlinear systems, exact evaluation of these probabilities is difficult. The proposed approach uses a scenario approximation. At each MPC update, generate S disturbance samples $\{w^{(s)}_{0:H-1}\}_{s=1}^{S}$ over the horizon H from the current uncertainty model, propagate the dynamics for each scenario under candidate controls, and enforce constraints for all scenarios, yielding a sampled approximation of the chance constraints. The resulting optimization is

$$\begin{aligned}
\min_{u_{0:H-1}} \quad & \sum_{k=0}^{H-1} \ell(x_k, u_k) + \ell_f(x_H) \\
\text{s.t.} \quad & x^{(s)}_{k+1} = f(x^{(s)}_k, u_k, w^{(s)}_k) \\
& p_{\text{min}} \le p_{\text{bh}}(x^{(s)}_k) \le p_{\text{max}} \quad \forall k, \forall s \\
& q_g(x^{(s)}_k) \le q_{g,\text{max}} \quad \forall k, \forall s \\
& u_{\text{min}} \le u_k \le u_{\text{max}}
\end{aligned} \tag{16}$$

where the state trajectories are scenario-dependent. Enforcing constraints for all scenarios corresponds to a conservative approximation of the chance constraint [23]. To match a desired violation probability, one can allow a fraction of scenarios to violate constraints by slack selection, or one can choose S according to probabilistic guarantees. In practice, an adaptive approach is used: early in an event, uncertainty is large and S is increased to maintain risk control; as uncertainty reduces through estimation, S can be decreased to reduce computation.

The scenario-based MPC can be computationally heavy if the underlying model is large. This motivates the differentiable surrogate introduced later. Before that, it is important to note how dissolution-aware augmentation changes the optimization landscape [24]. A free-gas-only model tends to interpret early pit gain as free-gas presence, which increases predicted surface gas flow and thus triggers constraints prematurely, leading to aggressive back-pressure or unnecessary flow reductions. A dissolution-aware model can represent that the same pit gain may correspond to swelling with limited free gas, reducing predicted near-term surface gas handling and allowing the controller to focus on maintaining bottomhole pressure within bounds with less disruption. This can enlarge the feasible set of controls and reduce control aggressiveness while maintaining safety probabilities.

However, dissolution-aware modeling can also increase predicted risk in different ways. Swelling increases return volume and can threaten pit capacity or surface flow limits. It can also alter frictional pressure drop by changing density and effective rheology [25]. The MPC must therefore include constraints on pit volume and return flow when relevant. These are included as additional chance constraints when the risk tolerance requires it:

$$\mathbb{P}\Big(V_{\text{pit}}(x_k) \le V_{\text{max}}\Big) \ge 1 - \epsilon_v \tag{17}$$

where V_{pit} is computed from volumetric balance including swelling effects. The interplay of these constraints can produce nontrivial trade-offs that a deterministic heuristic cannot systematically manage.

The optimization is solved in a receding horizon. Because the model is differentiable, gradient-based solvers can be applied, and warm-starting from the previous solution improves speed [26]. The next section develops a surrogate model that preserves differentiability and physical feasibility while reducing computational cost.

4. Differentiable Reduced-Order Prediction Model for Real-Time Optimization

Scenario MPC requires propagating dynamics many times per control update. Even a modest discretization of the annulus can be too slow when multiplied by scenario count and by solver iterations. A reduced-order model (ROM) is therefore constructed. The ROM must be accurate enough in the region of operation, must preserve physical bounds, and must be differentiable to enable efficient optimization [27]. The approach adopted here is a constraint-preserving latent dynamics model that combines physics-based structure with data-driven operator identification, designed for stable rollouts under varied control inputs.

Let $x \in \mathbb{R}^n$ denote the full discretized state. A latent representation $z \in \mathbb{R}^r$ with $r \ll n$ is introduced with an encoder E and decoder D such that $z = E(x)$ and $\tilde{x} = D(z)$. Because physical feasibility is critical, the decoder is structured rather than arbitrary. Components that must remain positive, such as densities, are represented in log coordinates; components that must remain bounded in $[0, 1]$, such as α_g and x_d, are represented through sigmoid maps. This yields a decoded state that is feasible by construction, which is essential because an infeasible state could violate constraints and mislead the optimizer.

The latent dynamics model approximates the discrete-time update:

$$z_{k+1} = F(z_k, u_k, \eta_k) \tag{18}$$

where η_k represents latent disturbances corresponding to uncertain parameters [28]. The function F is built as a low-order operator with learned coefficients but constrained to be stable. A practical choice is a polynomial operator with bilinear input coupling:

$$z_{k+1} = Az_k + Bu_k + c + \mathcal{B}(z_k, u_k) + \mathcal{Q}(z_k) \tag{19}$$

where $\mathcal{B}$ is bilinear and $\mathcal{Q}$ is quadratic, capturing the dominant nonlinearities while keeping evaluation fast. Coefficients are learned offline from simulated trajectories generated by the control-oriented full model across a range of controls, influx scenarios, and parameter variations.

Stability is encouraged by constraining the spectral radius of A and by penalizing rollout divergence during training. Rather than minimizing one-step prediction error, training minimizes multi-step rollout error under varied control sequences, because MPC requires accurate rollouts [29]. In addition, conservation-inspired constraints are enforced. For example, the total gas mass inventory, combining free and dissolved contributions, should evolve consistently with boundary fluxes and influx. A linear functional $m_g(x)$ measuring gas mass is computed from the decoded state. Training enforces that predicted changes in m_g match those implied by known source terms, preventing the ROM from inventing or destroying gas to fit short-term signals.

Thermodynamic closures are not learned from scratch [30]. The equilibrium mapping $x_d^\star(p, T)$ and compressibility factor $Z(p, T)$ are embedded as differentiable interpolation operators in the decoder. This hybrid approach reduces data requirements and improves generalization, because the ROM inherits the monotonic trends of thermodynamics. The ROM learns primarily the transport and coupling dynamics that determine how pressure and phase distribution evolve under controls.

A key requirement for scenario MPC is fast sensitivity computation. Because F is explicit and composed of differentiable operations, Jacobians $\partial z_{k+1}/\partial z_k$ and $\partial z_{k+1}/\partial u_k$ are available analytically. The chain rule through the decoder yields sensitivities of outputs with respect to controls, enabling gradient-based optimization [31]. This is critical for real-time feasibility because derivative-free optimization would require too many function evaluations.

Uncertainty enters the ROM through parametric perturbations. The uncertain parameters, such as friction multipliers, slip parameters, swelling coefficients, and mass-transfer rates, are represented by a low-dimensional vector θ. The ROM includes θ either by augmenting the latent state or by treating it as an exogenous input to F. A convenient representation is

$$z_{k+1} = F(z_k, u_k, \theta) + \omega_k \tag{20}$$

with ω_k a small additive disturbance capturing residual model mismatch [32]. Scenario generation draws θ from its current posterior distribution produced by the estimator, ensuring consistency between estimation and control.

To ensure that the ROM remains reliable, it is validated on held-out scenarios that include aggressive choke changes, pump ramps, and influx events with varied depths. Validation metrics focus on safety-relevant outputs such as bottomhole pressure and surface gas fraction, because MPC decisions depend on these. The ROM is also stress-tested for extrapolation within plausible ranges. Constraint-preserving decoding mitigates catastrophic failures such as negative densities or invalid phase fractions, which are common failure modes of unconstrained surrogates [33].

The ROM enables scenario MPC by reducing per-scenario rollout cost. However, the controller must still manage uncertainty in unknown influx parameters. This introduces dual control considerations: control actions affect both safety and information gain. The next section develops the uncertainty propagation and estimation mechanism and discusses how the MPC can incorporate information-seeking behavior without resorting to excessively complex formulations.

5. Uncertainty Propagation, Estimation, and Dual Control Considerations

During a suspected kick, the influx rate, onset time, and depth are uncertain [34]. In addition, model parameters controlling friction, slip, and dissolution kinetics are uncertain. The controller requires a state estimate and an uncertainty representation to generate scenarios and enforce chance constraints. The estimation problem is complicated by the fact that actuation alters the system response, and by the fact that dissolution can mask free-gas signals. The framework therefore integrates a sequential Bayesian estimator with the control loop, maintaining a consistent posterior over states and parameters [35].

Let $\hat{z}_k$ denote the latent state estimate at time k and let P_k denote an uncertainty representation. The estimator ingests measurements such as pit volume change, standpipe pressure, casing pressure, flow-in, and flow-out. The measurement function is $d_k = g(z_k, u_k) + \nu_k$, where ν_k is measurement noise and bias [36]. A practical estimator is an ensemble-based filter operating on the ROM, because the ROM is fast and can propagate ensembles in real time. The ensemble $\{z_k^{(i)}, \theta^{(i)}\}_{i=1}^{M}$ is propagated through the ROM under the applied control and is updated using measurement innovations. This yields a posterior ensemble that approximates $p(z_k, \theta \mid d_{0:k})$.

Bias in pit measurements and delays in flow sensors are handled by augmenting the state with bias and lag variables. This prevents the filter from forcing physical states to explain systematic errors. The augmented variables are modeled as random walks or first-order lags, which are identifiable when sufficient excitation exists. In a managed pressure environment, excitation often comes from control actions. This creates an opportunity: control actions can be designed to not only maintain safety but also to improve identifiability of influx and dissolution states [37].

Dual control formalizes this opportunity: one seeks control inputs that balance exploitation, meaning maintaining safe and efficient operation, and exploration, meaning reducing uncertainty. Exact dual control is intractable for nonlinear, constrained systems. The paper adopts a pragmatic approach: include an information-seeking term in the MPC objective that encourages controls that reduce predicted uncertainty in key variables, subject to safety. A simple surrogate for information gain is the predicted variance of influx-related outputs or the trace of a predicted covariance of selected parameters. Using linearized uncertainty propagation, one can compute an approximate covariance update under candidate controls: [38]

$$P_{k+1} \approx A_k P_k A_k^\top + W_k \tag{21}$$

where A_k is the Jacobian of the ROM dynamics and W_k is process noise covariance. Measurement updates reduce covariance depending on observability. The MPC objective can include a term such as $\mathrm{tr}(P_{k+H})$ for selected blocks corresponding to influx rate parameters. This encourages control sequences that improve observability without explicitly optimizing mutual information.

The dissolution-aware augmentation matters for observability. If pit gain is influenced by both swelling and free-gas expansion, then changes in choke pressure that alter pressure distribution can change dissolution equilibrium and thus modulate pit gain in a way that reveals whether gas is predominantly dissolved or free [39]. A controller that applies a small, safe modulation in backpressure can create a diagnostic signature: if pit gain responds rapidly to pressure modulation, dissolved gas is likely significant; if it does not, free gas may dominate. The MPC framework can exploit this by selecting control sequences that are informative while ensuring chance constraints on bottomhole pressure remain satisfied.

Uncertainty propagation into chance constraints is performed through scenarios drawn from the posterior ensemble. This ensures that scenario variability reflects the current belief about influx and parameters, including dissolution kinetics. When uncertainty is large, the controller becomes more conservative, but dissolution-aware modeling can reduce unnecessary conservatism by preventing the ensemble from overestimating free-gas handling risk when swelling explains part of the observations.

A practical issue is computational load: scenario MPC with large S and large M can be heavy [40]. The framework therefore uses a two-layer approach. The estimator maintains an ensemble of size M for posterior approximation. For control, a smaller scenario set of size S is generated by resampling or by selecting representative particles that cover posterior extremes relevant to constraints. This reduces computation while maintaining safety coverage. In addition, scenario allocation can be adaptive: allocate more scenarios to uncertainties that most influence constraints, such as influx rate and friction, and fewer to weakly influential parameters [41].

Constraint tightening is also used as a complementary technique. When approximate linear-Gaussian propagation is adequate over short horizons, chance constraints can be converted into deterministic tightened constraints by adding a margin proportional to predicted standard deviation. For example, a one-sided chance constraint for bottomhole pressure can be approximated by

$$\mu_{p,k} - \gamma_p \sigma_{p,k} \geq p_{\min} \tag{22}$$

where $\mu_{p,k}$ and $\sigma_{p,k}$ are predicted mean and standard deviation and γ_p is a quantile factor. Tightening reduces the number of scenarios needed when distributions are roughly unimodal. Scenario methods are reserved for strongly nonlinear regimes where tightening may be inaccurate [42]. The controller can switch between these modes based on nonlinearity indicators, such as mismatch between linearized and nonlinear rollouts.

The combined estimator-controller loop thus maintains a consistent belief state and computes controls that keep risk within bounds. The final technical section evaluates this architecture in representative drilling scenarios and compares dissolution-aware control against a baseline free-gas-only model predictive controller.

6. Numerical Evaluation in Dissolution-Dominated and Free-Gas-Dominated Regimes

The numerical experiments evaluate both feasibility and risk performance of the proposed chance-constrained dissolution-aware MPC. A simulated managed pressure drilling system is constructed with a representative annulus geometry, compressible gas behavior, and oil-based mud properties [43]. The control horizon is chosen to capture the main transient response of bottomhole pressure and surface returns, and the controller is updated at a cadence consistent with typical surface measurements.

Two classes of scenarios are examined. The first class is dissolution-dominated: solubility is high and the mass-transfer rate is sufficiently large that a substantial fraction of influx gas dissolves over operational time scales. In these scenarios, early pit gain is driven primarily by swelling, and free-gas surface arrival is delayed. The second class is free-gas-dominated: solubility is low or mass-transfer is slow, and free gas persists and migrates upward, producing more direct surface gas handling risk. In both classes, the influx onset time and rate profile are uncertain, and friction parameters vary to represent changing cuttings loading and rheology [44].

The controller is compared against two baselines. The first baseline is a deterministic MPC using a nominal model without chance constraints. The second baseline is a chance-constrained MPC that uses a free-gas-only model with no dissolved state. All controllers enforce the same actuator bounds and aim to maintain bottomhole pressure near a reference within the window. Performance is evaluated by the frequency of constraint violations, the average deviation from reference pressure, the magnitude and variability of control actions, and the predicted and realized surface gas handling [45].

In dissolution-dominated scenarios, the dissolution-aware controller shows a consistent feasibility advantage. The free-gas-only chance-constrained controller interprets early pit gain as free gas and predicts earlier surface gas arrival. As a result, it increases backpressure aggressively and may reduce pump rate, which can increase equivalent circulating density and approach fracture constraints. This can produce infeasibility when attempting to satisfy both bottomhole pressure upper bounds and assumed surface gas constraints. The dissolution-aware controller, by contrast, predicts that much of the influx is dissolved and thus reduces near-term surface gas risk [46]. It maintains backpressure closer to the minimum required for bottomhole pressure safety and avoids unnecessary pump reductions. The result is fewer near-fracture excursions and reduced actuation variability while maintaining the prescribed probability of bottomhole pressure containment.

In free-gas-dominated scenarios, both chance-constrained controllers behave similarly in their early response, because pit gain and pressure signals align with free-gas expansion and migration. The dissolution-aware controller may still differ in late-stage predictions if dissolution becomes relevant at depth as pressure increases, but the dominant hazard is free gas. In these scenarios, the dissolution-aware controller does not compromise safety; rather, it provides comparable constraint satisfaction while maintaining a more accurate posterior over dissolved fraction, which can matter for surface handling prediction when pressure modulation occurs.

A third set of scenarios introduces confounding operational transients such as pump ramp-down and choke perturbations without influx [47]. These scenarios test false alarm tendencies. The deterministic controller can overreact if it treats deviations as evidence of influx, especially if it uses a model that does not represent sensor bias or transient friction changes. The chance-constrained controllers, which propagate uncertainty and incorporate estimation, are less prone to overreaction because they do not treat single-sensor anomalies as definitive. Dissolution-aware augmentation further reduces spurious conclusions about free gas when pit gain is affected by swelling-like signatures caused by temperature shifts or compressibility changes, though these are modeled only approximately.

The result is reduced unnecessary actuation in non-kick scenarios, which is important because excessive actuation can itself destabilize operations [48].

The experiments also evaluate risk certificates. For each controller, the predicted probability of violating bottomhole pressure bounds and of exceeding surface gas handling limits is computed from scenarios. The dissolution-aware controller provides tighter certificates in dissolution-dominated cases, meaning it can meet the same risk thresholds with less conservative actions. This is not because it ignores risk, but because it partitions pit gain into dissolved and free components, reducing the predicted free-gas hazard when appropriate. Conversely, when the posterior indicates that dissolution is insufficient to buffer the influx, the dissolution-aware controller becomes appropriately conservative and may increase backpressure or reduce circulation to delay surface arrival [49].

Sensitivity to mass-transfer rate uncertainty is examined. When the true mass-transfer rate is slower than assumed, a controller that is overly confident in dissolution buffering could underpredict free-gas hazard. The proposed framework avoids this by maintaining κ_d as an uncertain parameter in the posterior. Scenarios include slow and fast κ_d draws, and chance constraints must hold across these draws. As a result, the controller remains conservative when the posterior permits slow dissolution, and it becomes less conservative only when measurements reduce that uncertainty [50]. This demonstrates the importance of consistent uncertainty propagation: dissolution-aware modeling is beneficial only when accompanied by uncertainty representation, because the dissolution kinetics are not perfectly known.

Computational performance is reported qualitatively in terms of real-time feasibility. The differentiable ROM enables scenario rollouts and gradient computations within a budget consistent with control update times, whereas full-model rollouts would be too slow for scenario counts required by chance constraints. Warm-starting and the structured operator form reduce iteration counts. The trade-off between scenario count and conservatism is visible: higher scenario counts provide stricter risk control but increase computation. Adaptive scenario allocation mitigates this, focusing samples on the most influential uncertainties [51].

Overall, the numerical suite supports the claim that dissolution-aware state augmentation can improve the feasibility and efficiency of risk-aware managed pressure drilling control in regimes where swelling masks early influx signatures. The gains are largest when the operational window is narrow and when early pit gain is not a reliable indicator of free-gas hazard. The approach remains compatible with free-gas-dominated regimes and does not reduce safety when dissolution is weak, provided that dissolution parameters are treated as uncertain and are updated through estimation.

7. Conclusion

This paper presented a chance-constrained stochastic model predictive control architecture for managed pressure drilling that explicitly accounts for gas dissolution and mud swelling through thermodynamic state augmentation. The work reframed influx management as a risk-aware control problem rather than a detection-only problem, emphasizing that actuation alters observability and that uncertainty must be propagated into safety constraints [52]. A control-oriented one-dimensional annular model was developed with a bounded dissolution closure and swelling-modified liquid properties, discretized in a manner preserving physical feasibility for embedding in optimization. Chance constraints on bottomhole pressure, surface gas handling, and pit volume were approximated through scenario propagation consistent with a sequential Bayesian estimator. A differentiable reduced-order prediction model was introduced to enable real-time optimization across scenarios, preserving positivity and boundedness by construction and embedding thermodynamic equilibrium lookups for robustness.

The numerical experiments indicated that dissolution-aware augmentation can reduce spurious conservatism in dissolution-dominated regimes by preventing early swelling-driven pit gain from being misinterpreted as immediate free-gas hazard, thereby improving feasibility and reducing unnecessary actuation while maintaining prescribed violation probabilities. In free-gas-dominated regimes, the framework preserved safety and offered comparable performance, with uncertainty-aware buffering preventing overconfident reliance on dissolution kinetics [53]. The results also highlighted that the benefits of dissolution-aware modeling depend on representing uncertainty in dissolution parameters and property correlations; without uncertainty propagation, the same mechanisms that reduce conservatism can create hidden risk.

Limitations remain. The control-oriented model simplifies multiphase hydrodynamics and temperature evolution, and its adequacy depends on calibration to the operational envelope. Scenario MPC can be computationally demanding if uncertainty is high and horizons are long, motivating further work on tighter risk approximations and faster solvers. Future extensions include richer thermal coupling, improved slip closures informed by downhole sensing, and integration with operational decision support so that risk thresholds and objectives reflect context-dependent priorities. Within these limits, the proposed framework provides a coherent method for combining dissolution-aware physics with probabilistic safety guarantees in real-time managed pressure drilling control [54].

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